A Customized Lease Contract for Fleet

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ABSTRAK

In this paper we study a customized two-dimensional lease contract for a fleet of public transport, such as buses, shuttle etc. The lease contract are characterized by two parameters – age and usage – which define a two-dimensional region. However, we use one dimensional approach to model these age and usage of the fleet. The contract is classified into two categories usage rate with different operating conditions and then different preventive maintenance (PM) schedules are applied to the contract in the different categories. Periodic preventive maintenance is considered to meet different contract requirements to ensure the reliability of the fleet. Fleet with high utilization rates could have satisfactory reliability by performing more frequent PM, while those with low and medium utilization rates could reduce the cost of PM within the lease contract. We further assume that there will be three different usage pattern of the buses, i.e. low, medium, and high pattern of the usage rate. In many situations it is often we face a blur boundary between the adjacent patterns. The result show that the classification of fleet into different usage pattern cannot only be beneficial by providing an effective mechanism to reduce the lease costs, but also be a valuable competitive marketing strategy for lease provider.

Kata kunci— Two-dimensional lease contract, preventive maintenance, minimal repair, expected maintenance cost, fuzzy.

I. INTRODUCTION

In many areas of industry, outsourcing for maintenance of vital equipment becomes prevalent. Especially when the equipment is sophisticated and need a special expertise to maintain, such as modern buses in a public transport service system. Hence an economical way to carry out maintenance is to outsource the maintenance works to an external agent. The agent can do a partial or full coverage of the maintenance actions, such as Preventive Maintenance (PM) or/and Corrective maintenance (CM).

Maintenance involving two parameter – i.e. age and usage for N repairable units have received attention in the literature (see Husniah et al. (2014, 2017) and Iskandar et al. (2014)). They have purposes an incentives to increase the equipment’s performance beyond target. In contrast to all works previously, Husniah et al. (2015) studied a lease contract with periodic PM which considers penalty cost to shield an agent against over claim. Two dimensional approach has been used in many warranty papers which has been pioneered by Iskandar et al. (2005) and Iskandar and Murthy (2003). As pointed by Huang et al. (2013) two-dimensional approach has many benefits, such as the increase of competitive advantage and customer loyalty (Shahanaghi et al., 2013). To increase the realism of the model, some complexities can easily incorporated into two-dimesional approach, such as the types of customer. As an example, the authors in Huang et al. (2015) developed a two-dimensional warranty policy by considering two types of customers, i.e., customers with the adoption of policies concerning the warranty time or usage limit. Other type of customers can be characterized by the pattern of their usage rate. To date, a two-dimensional lease contract which considering the types of customer usage rate is still limited. Only few literatures discussing this issue despite it is an important factor, since different types of customer give different optimal cost of the contract. The authors in Husniah et al. (2015) devised a two-dimensional lease contract model and discussed a contract for the case where the owner has a fleet consists of N units. They consider two types of customer usage rate, i.e. medium and high usage rates. They used a crisp set membership of the usage rate level. In their hypothetical example, the usage rate of 1.35 is regarded as a medium usage rate while the usage rate of 1.75 is regarded as a high usage rate. It is not clear
how to categorize the usage rate in between those values, e.g. is 1.55 belong to medium or high usage rate?

In reality, it is difficult to decide a membership of a value which lies just around a boundary. In this paper we extend the model in Husniah et al. (2015) to consider the vagueness of membership using the fuzzy set theory. The use of fuzzy set theory (including fuzzy numbers, fuzzy logic, fuzzy knowledge, and fuzzy decision rules) is not new in maintenance modeling. Even the authors in Strackeljan, J. and Weber, R. (1999) emphasized that maintenance is among the areas where fuzzy sets have been applied intensively. Some of the literatures are highlighted in the following.

Wolkenhauer (2001) gave an example of the application of fuzzy logic in maintenance decision making by considering the downtime of machines and the frequency of faults. Sergaki and Kalaitzakis (2002) used a fuzzy knowledge based method for maintenance planning and applied it in a power system. Fuzzy logic is used to obtain adaptive preventive maintenance (Yuniarto and Labib, 2006), scheduling preventive maintenance (Fouad and Samhouri, 2011), imperfect maintenance (Hennequin et al., 2009), and recently it is used in Segura et al. (2016) to model the imperfections of maintenance actions due to operators, in which they argued that the level of worker’s skill is not crisp, so that the problem is well suited by the fuzzy set theoretical approach. Khanlari et al. (2008) used fuzzy rules to interpret linguistic variables for determination of maintenance priorities, in which verbal expressions are among important factors in determining the priorities. Using this approach, the verbal expressions are quantified and used in decision making, which otherwise cannot be explicitly analyzed. To sum up, the authors in Carvalho, et al. (2015) argued that in reality there are always uncertainty of costs and reliability parameters in maintenance problems. This will rise problem if it is omitted by the model. They developed a maintenance model to accommodate the uncertainty by representing these parameters as fuzzy numbers. They applied the model to the wind turbine pitch control device. They further argues that the model would facilitate managers to make their decisions based on a richer set of information. As pointed earlier, in this paper we extend the model in Husniah et al. (2015) to consider the vagueness of membership boundary of customer usage rate using the fuzzy set theory. We look for the optimal price for the agent and the optimal option for the owner, in which the usage rate is known to be varying according to low, medium, and high level. The paper is organized as follows. Section 2 will give the derivation of the model, Sections 3 and 4 present the model analysis and the numerical examples. Conclusion and further research are given in Section 5.

II. MODEL FORMULATION

In this section we derive maintenance model of a fleet of buses own by a government public transport service Indonesia, known as DAMRI. To begin the derivation we define the following notations that will be used in model formulation:

\[ \Omega_t = [0, \Gamma_0) \times [0, U_0) \] : Lease contract coverage

\[ \Delta_y \] : Preventive maintenance level

\[ X_i \] : Downtime caused by the \( i \)-th failure and waiting time

\[ D(t) \] : Total downtime in \( (0,t] \)

\[ F(t) \] : Distribution function of downtime

\[ \theta \] : Down time target

\[ Y \] : Usage rate

\[ C_r \] : Repair cost

\[ C_0 \] : Preventive maintenance cost

\[ C_p \] : Degree of preventive maintenance cost

\[ C_{pr} \] : Penalty cost per unit of time

\[ J \] : Expected lease contract cost

\[ r(t), R(t) \] : Hazard, and Cumulative hazard functions associated with \( F(t, \alpha_i) \)

\[ Z \] : Number of fleet

A. Lease Contract Policy

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We consider that DAMRI operates a number of buses and lease the bus with full covered PM to provide more protection. As in Husniah et al. (2015) the lease contract is offered with a two-dimensional lease contract with the lease characterised by a rectangle region \( \Omega_e = [0, \Gamma_0] \times [0, U_0] \) where \( \Gamma_0 \) and \( U_0 \) are the time and the usage limits (e.g. the maximum coverage for \( \Gamma_0 \) (e.g. 1 year) or \( U_0 \) (e.g. 50.000 km). All failures under lease contract are rectified at no cost to the lessee. For a given usage rate \( y \) of the, the lease contract ceases at \( \Gamma_y = \Gamma_0 \) for \( y \leq U_0 / \Gamma_0 \), or \( \Gamma_y = U_0 / y \) for whichever occurs first.

As the lease contract is full coverage (PM and CM), then a penalty cost incurs the lessor if the actual down time falls above the target \( (\delta) \). If \( D \) is down time (consisting repair time and waiting time) for each failure occurring during the contract, then the lessor should pay a penalty cost when \( D > \delta \). The amount of the penalty cost is assumed to be proportional to \( \Delta = D - \delta \). The penalty cost \( (\mathcal{C}_p) \) is viewed as a penalty given by the lessor. The decision problem for the lessor is to determine the optimal number of PM and degree of maintenance level such that to minimize the expected cost. Furthermore, we consider the case where age, usage and operating condition where the bus is operated as major factors to influence failure. Here, the accelerated failure time (AFT) model is an appropriate model to be used as it allows to incorporate the effect of the three major factors on degradation of the truck. If the distribution function for \( T_0 \) is given by \( F_0 (T, \alpha_0) \), where \( \alpha_0 \) is the scale parameter, then the distribution function for \( T_y \) is the same as that for \( T_0 \) but with a scale parameter given by \( \alpha_y = (\gamma_0 / y)^{y} \alpha_0 \) with \( \gamma \geq 1 \). Hence, we have \( F(t, \alpha_y) = F((\gamma_0 / y) t, \alpha_0) \). The hazard and the cumulative hazard functions associated with \( F(t, \alpha_y) \) are given by \( r_y (t) = f(t, \alpha_y) / (1 - F(t, \alpha_y)) \) and \( R_y (t) = \int_0^t r_y (s) \, ds \) respectively where \( f(t, \alpha_y) \) is the associated density function.

Let \( Y \) be a usage rate of the bus. We consider that \( Y \) varies from customer to customer but is constant for a given customer (or a given equipment). \( Y \) is a random variable with density function \( g(y), 0 \leq y < \infty \). Conditional on \( Y = y \), the total usage \( u \) at age \( t \) is given by \( u = yt \). Within the lease coverage, a lease contract ends at \( \Gamma_y = \Gamma_0 \) for given usage rate \( y \). Two cases need to be considered–i.e. (i) \( y \leq \gamma \) and (ii) \( y > \gamma \).

### Preventive Maintenance Policy:

We define periodic PM policy for a given \( Y = y \). PM policy for a given \( y \) is characterised by single parameter \( r_y \). The equipment is periodically maintained at \( k r_y \). Any failure occurring between pm is minimally repaired. Note \( (k+1)r_y = T_0 \) where \( k \) is an integer value. For a given usage rate \( y \), the effect of each PM action \( \Delta_j = \Delta_{j+1} = \Delta \) then \( r_y(t) = r(t) - j \Delta \). If any failure occurring between pm is minimally repaired, then expected total number of minimal repairs in \( (\{t_{j-1}, t_j\}, 1 \leq j \leq k_y + 1) \) is given by \( N = \sum_{j=1}^{k_y} \int_{t_{j-1}}^{t_j} r_j(t') \, dt' = R(\Gamma_0) - \sum_{j=1}^{k_y} (\Gamma_0 - j \Delta) \Delta_j \) for \( t_{j-1} - t_j = \tau_j \). As the lease contract is full coverage (PM and CM), then a penalty cost incurs the OEM if the actual down time falls above the target \( (\delta) \). The penalty cost \( (\mathcal{C}_p) \) is viewed as a penalty given by the OEM. The decision problem for the OEM is to determine the optimal price structure and maintenance level such that to minimize the expected cost.

### III. MODEL ANALYSIS

Suppose that there are \( k \) \( (k > 1) \) failed buses will be served by the OEM with a single service channel using the first come, first served rule. We consider a situation where the lessor incurs repair cost for each failure and PM cost. For a larger coverage of lease contract e.g. for maximum 5 years or 250.000 km, it would require more than one PM for reducing the maintenance cost. In the proposed lease contract, the lessor expected total cost consists of PM cost, repair cost and penalty cost. The lesor incurs penalty cost when the down time caused by a failure exceeds the predetermined target.
Suppose that there are \( k \) units failed equipment will be served by the lessor with single service channel where the first come, first served. Hence, there is queue which the model formulation is identical to a Markovian queue. The arrival rate of failed equipment is \( \lambda = (Z - k)\bar{\lambda} \) for \( 0 \leq k \leq Z \), where \( Z \) is number of equipment population and \( \bar{\lambda} \) is failure rate. The service rate is \( \mu \). According to Husniah, dkk.(2015) the steady state density function for \( Y_j \) (waiting time for truck \( j \)) is

\[
f(y) = \sum_{k=0}^{Z-k} P_k \mu e^{-\mu} \frac{(\mu y)^k}{k!} \]

where \( P_k, \) \( k = 1, 2, ..., Z - 1 \) given by the ratio \( P_k = P_{k+1} / P_k \) and \( P_k = (Z-k)(\lambda/\mu)^k (Z!/((Z-k)!)) \). \( \bar{\mu} \) is estimated by the mean value of failure intensity, \( \bar{\lambda} \). The expected value of \( Y_j \) is \( E[Y_j] = \int_{0}^{\infty} y f(y) dy = \sum_{k=0}^{Z-k} P_k \frac{(k+1)}{\mu} \). Here, two cases are considered – case (i) \( y \leq \gamma \) and case (ii) \( y > \gamma \). Hence, for \( y \leq \gamma \), the total expected cost of the lessor is

\[
E[\pi_j] = Z(\gamma + C_g \tilde{G}(\gamma) N(k_j, \tau_j)) \]

with \( C_g \tilde{G}(\gamma) \) is defined as \( C_g \tilde{G}(\gamma) = C_g \int_{0}^{\gamma} (y - \gamma) g(y) dy \).

For case \( y > \gamma \), the expected cost of lessor is given by replacing \( \Gamma_0 \) with \( \Gamma_y \).

### IV. NUMERICAL EXAMPLE

In this numerical example we vary the usage rate \( \gamma \). For a given usage rate \( y \), the failure distribution is given by the Weibull distribution. Let the parameter values be as follows, \( \alpha_0 = 1(\text{years}) \), \( \beta = 2.25 \), \( \Gamma_0 = 2(\text{years}) \), \( U = 2(10^9 \text{Km}) \), \( \gamma = U/W = 1 \), \( \rho_0 = 1 \), \( \alpha_0 = 100 \), \( \theta_0 = \theta_c = 0.5 C \), \( s = 80 \text{ (hours)} \) or 4 \text{ (days)} \) \( \) or \( \theta_0 = 3 \) the service rate is \( 10^5 \text{ 2025} \$ \) the down time is given by the exponential distribution with \( 1/\lambda = 1/300 \text{ (years)} \). The road condition is reflected by \( \rho = 1.2, 2.0 \) and 2.2 corresponding to light incline, high incline and very hilly, respectively. Table \( 1 \) shows optimal number of PM and improvement level of lease contract with three usage types – low (1.0 \( \leq y < 1.2 \) ), medium \( (1.2 \leq y < 1.4) \) and heavy (1.4 \( \leq y < 1.8 \)).

Now if we consider one of the input, i.e. the usage rate, is a fuzzy number rather than a crisp number, then the output certainly would be a fuzzy number. For example, the crisp number \( y = 1.30 \) in Table \( 1 \) now should be considered as a fuzzy number, let say it is represented as a triangular number \( y = (1.20, 1.30, 1.40) \). There are several ways to treat a fuzzy number as an input to a function. Lee (2005) pointed out that fuzzy function can be classified into three groups according to which aspect of the crisp function the fuzzy concept was applied, namely crisp function with fuzzy constraint, crisp function which propagates the fuzziness of independent variable to dependent variable, and function that is itself fuzzy, this fuzzifying function blurs the image of a crisp independent variable. In this paper we use the second approach, in which a crisp function propagates the fuzziness of independent variable to dependent variable.

<table>
<thead>
<tr>
<th>( \rho ) = 1.2</th>
<th>( \rho ) = 2.0</th>
<th>( \rho ) = 2.2</th>
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<tbody>
<tr>
<td>( \gamma )</td>
<td>( k^* )</td>
<td>( \tau^*_j )</td>
</tr>
<tr>
<td>Low</td>
<td>1.00</td>
<td>1.70</td>
</tr>
<tr>
<td>Medium</td>
<td>1.10</td>
<td>1.72</td>
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Figure 1.a shows the graphical image of the triangular fuzzy number $\tilde{y}=(1.20,1.30,1.40)$ representing the medium rate and the multimodal fuzzy numbers $\tilde{y}=(1.00,1.20,1.40)$, and $\tilde{y}=(1.20,1.40,1.80)$ representing the low and high rate, respectively. Figures 1.b to 1.d show the resulting expected lease cost for operating condition equals to 1.2, 2.0, and 2.2 respectively. Qualitatively the resulting costs are similar except the value is higher for the higher usage rate.

![Graphical Image](image)

V. CONCLUSION

In this paper we give an example on how to accommodate the fuzziness of a parameter in determining the optimal fleet number and the optimal strategy for maintenance the fleet. In general, for a given crisp value $y$ and a fixed reliability level, the optimal expected profit decreases as the usage rate $y$ increases. This is become more prevalent in the presence of fuzziness, by observing the output of the low, medium, and high usage rates, which gives the relatively high, medium, and low numbers of fleet, profit, and price of the optimal option. In reality, the usage rate of a bus increases due to a longer travelled distance from a station to other station. This is as expected since the increasing in $y$ causes the failure rate to increase and this in turn increases the number of failures under the lease contract. It is best of the lessor to perform more PM activities for buses with low and average utilization rates, in order to reduce the repair costs. On the other hand, if performing a PM activity becomes more expensive, the lessor would have to bear more expenditure, and thus reducing the number of PM activities would become a better option.
choice. Different shape of membership can be explored to investigate the sensitivity of the results presented in this paper.

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